Handout #23
March 6, 2017

Section Handout 8

Problem One: Bits, Bytes, Characters, and Strings

In C++, what is the difference between `0`, `'0'`, and `"0"`? Does the statement `0 == '0'` compile? If so, does it evaluate to `true` or `false`? Does the statement `'0' == "0"` compile? If so, does it evaluate to `true` or `false`?

Problem Two: Huffman Encoding

Draw the Huffman encoding tree you'd get for the string "avicenna's canon" and write what that string looks like when encoded with that tree. Don't forget the pseudo-EOF character!

Problem Three: Scrambled Hashes

Below are three descriptions of hash functions that can be used to produce hash codes for English words. Each of these functions has a problem with it that would make it a poor choice for a hash function. For each of the hash functions, describe what the weakness is.

- Hash function 1: Always return 0.
- Hash function 2: Return a random `int` value.
- Hash function 3: Return the sum of the ASCII values of the letters in the word.

Problem Four: Rehashing

In lecture, we mentioned that in order to keep the runtime of a hash table low, we periodically need to ***rehash*** the elements by doubling the number of buckets and moving the elements from the old table into the new one. The `OurHashSet` type has the following private fields:

```
Vector<Vector<std::string>> buckets;
int numElems;
```

Here's the code for the insert function:

```
void OurHashSet::add(const string& value) {
    /* Determine the bucket to jump into. */
    int bucket = hashCode(value) % buckets.size();

    /* If this element is already present, we don't need to do anything. */
    for (string elem: buckets[bucket]) {
        if (elem == value) return;
    }

    buckets[bucket] += value;
    numElems++;
}
```

Implement a function `OurHashSet::rehash()` that performs a rehash, then update the above code from the `OurHashSet::add` function to rehash the table whenever the load factor (the ratio of the number of elements to the number of buckets) is two or greater.

Problem Five: Custom Hash Codes

Suppose that you have the following struct representing a city:

```
struct City {
    string name;
    int population;
};
```

To make a HashSet<City>, you need to implement the following two functions:

```
int hashCode(const City& city);
bool operator== (const City& lhs, const City& rhs);
```

This first function computes a hash code for the given city, and the second allows us to compare two cities against one another using the == operator to tell whether they're equal.

Implement the hashCode function by following this template, which does a good job in general of distributing elements:

```
int hashCode(const Type& value) {
    int result = hashCode(value.field1);
    result = 31 * result + hashCode(value.field2);
    result = 31 * result + hashCode(value.field3);
    …
    result = 31 * result + hashCode(value.fieldN);
    return result & 0x7FFFFFFF;
}
```

Curious why this works? Check out the textbook!

To implement the **operator**== function, write code that checks whether each field of the two cities match one another.

Problem Six: A Classic Job Interview Question

Here's another classic job interview question that you are now equipped to answer. ***Useful hint: the answer to a ton of technical job interview questions is "use a hash table." I'm not kidding.***

You are given a list of integers. Write a function to find if any two of those integers add up to exactly 137. Your solution should be as fast as possible. Aim to solve it with an algorithm that, on average, runs in time $O(n)$.

As a follow-up, consider the following problem, which is called the 3SUM problem: given a list of integers, determine whether any *three* of those integers sum up to exactly 137 (you're allowed to choose the same number multiple times if you'd like). See if you can solve this in time $O(n^2)$.

Problem Seven: Linear Probing

There are a lot of different ways to build a hash table, each of which is designed to address collisions in a different manner. The type of hash table we used in class uses a system called ***closed addressing:*** any two items that collide with one another are put into the same bucket. There's another type of hashing called ***open addressing*** that in practice tends to be much, much faster. The simplest type of open addressing (which happens to be the very first sort of hash table ever invented!) is called ***linear probing***.

In linear probing hashing, the hash table consists of an array of table slots. To insert an element, we compute it hash code, then look in the table slot. We then do the following:

- If the slot is empty, then we just put the element into that table slot.
- If the slot is full and it holds the element we're inserting, there's nothing to do.
- Otherwise, move to the next slot in the table, wrapping around as necessary, and repeat this process.

Intuitively, you can think of linear probing hashing this way: each element has a slot that it would like to be in (the slot corresponding to its hash code), and if it can't fit there, it scoots over until it finds a free position. Similarly, to look up an element in a linear probing hash table, you'd compute the hash code for that element, then look in the slot in the table it corresponds to. From there, we'd do the following:

- If the slot is empty, then the element is definitely not present in the table.
- If the slot is full and the element there is the element in question, we've found it!
- Otherwise, move to the next slot in the table, wrapping around as necessary, and repeat this process.

Linear probing hash tables need to keep their load factors relatively low, since otherwise the table starts to have really long runs of elements that degrade lookup times. Typically, you'd pick a load factor like 0.75 and rehash if there are more than that many elements in the table.

Implement the following interface for a class that uses linear probing to implement a hash table:

```
class LinearProbingTable {
public:
    LinearProbingTable();

    bool contains(int value) const;
    void add(int value);

    int size() const;
    bool isEmpty() const;
public:
    /* Your call! */
};
```

You can assume that all the integers that will be stored in the table are nonnegative, so you can reserve negative values as sentinels to indicate "not present." In practice, linear probing hash tables tend to *dramatically* outperform the closed-addressing system we talked about in class. Take CS166 for details!

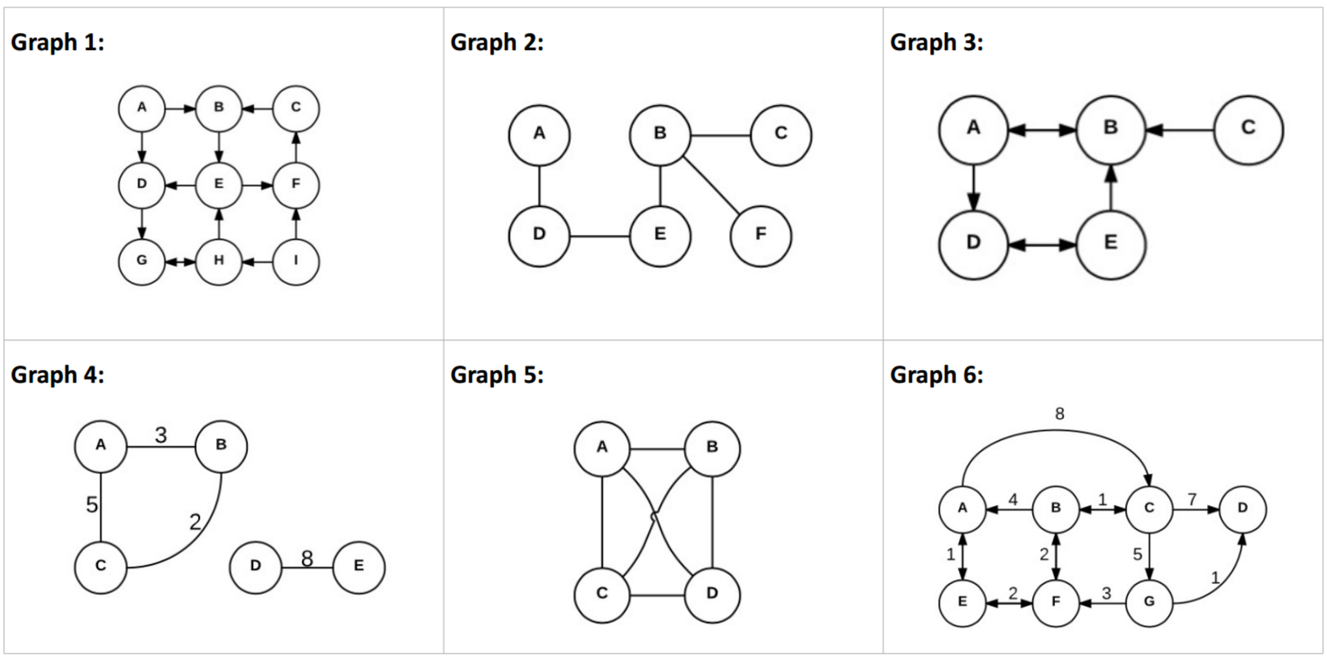
Problem Eight: Word Ladders as a Graph Search

In Assignment 2, you implemented an algorithm to find the shortest word ladder between two words. We introduced that problem as an example of breadth-first search, which you've just seen in its most general form. Describe how to model the Word Ladders problem as doing a breadth-first search in an appropriately-constructed graph. What are the nodes? What are the edges?

Problem Nine: Graph Searches

Below are six graphs. For each graph, say whether that graph is directed or undirected. Then, run a depth-first search and a breadth-first search starting at node *A* in each graph. When doing the depth-first search, at each point in time, if you have multiple choices of which node to visit next, choose the one that's alphabetically first. When doing the breadth-first search, similarly choose to enqueue nodes in alphabetical order if there's a tie. For the BFS, at each point show the node dequeued out of the queue and the state of the queue after processing that node.

Graphs 4 and 6 have weighted edges. Trace through an execution of Dijkstra's algorithm on those graphs. Show which nodes are in the priority queue at each point in time and what their priorities are, along with the final distances you compute to each node.

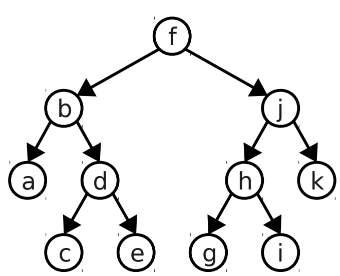


Problem Ten: Breadth-First Search and Binary Search Trees

All trees are graphs (though not all graphs are trees), so it's possible to use BFS to traverse a tree. Write a function

```
void breadthFirstSearch(Node* root);
```

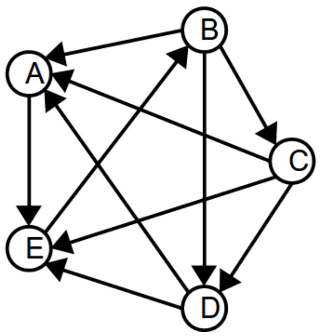
that accepts as input a pointer to the root of a binary search tree, then prints out all the nodes in the tree using a breadth-first search. What will the output of your function be on the following tree?



Under what circumstances will your function list all the nodes in a BST in sorted order?

Problem Eleven: Tournament Winners

A ***tournament*** is a graph representing contest among *n* players. Each player plays a game against each other player, and either wins or loses the game (let's assume that there are no draws). Each player is represented by a node, and each directed edge represents the outcome of a game, with the winner pointing to the loser. For example, in the tournament to the left, player A won her game against player E, but lost against players B, C, and D.



A ***tournament winner*** is a player in a tournament who, for each other player, either won her game against that player, or won a game against a player who in turn won his game against that player (or both). For example, in the graph on the left, players *B*, *C*, and *E* are tournament winners. However, player *D* is ***not*** a tournament winner, because he neither beat player *C*, nor beat anyone who in turn beat player *C*. Although player *D* won against player *E*, who in turn won against player *B*, who then won against player *C*, under our definition player *D* is ***not*** a tournament winner. *(Make sure you understand why!)*

There's a cool theorem about tournaments that says that every tournament must have at least one winner (curious why? take CS103!) Write a function

```
Set<String> winnersOf(const Map<string, Set<string>>& tournament);
```

that takes as input a graph representing a tournament and returns a set of all the tournament winners in that tournament.

Problem Twelve: Tournament Victory Chains

Here's another cool fact about tournaments (as defined above). In any tournament, there's always some way to line up all the people in the tournament so that each person won her game against the person immediately to her right. For example, in the tournament shown above, we can line up the players as *D*, *A*, *E*, *B*, *C*, since *D* beat *A*, *A* beat *E*, *E* beat *B*, and *B* beat *C*. This is sometimes called a ***victory chain***.

There's a very cool recursive algorithm for finding such a victory chain. If the tournament has no players, then the empty list of players is a victory chain. Otherwise, choose a person *p* and split the tournament into two subtournaments consisting of the players who beat *p* and the players who lost to *p*, respectively. Recursively get victory chains for those subtournaments. The overall chain can then be formed by starting with the chain for the subtournament of folks who won against *p*, then going to person *p*, then finishing with the chain for the subtournament of folks who lost against *p*. (Do you see why this works?)

Implement a function

```
Vector<string> victoryChainFor(const Map<string, Set<string>>& tournament);
```

that takes in a tournament and returns one of its victory chains.

Problem Thirteen: Eccentricities

The ***distance*** between two nodes in a graph is the length of the shortest path between them. The ***eccentriciy*** of a node in a graph is the maximum distance between that node and any other node in the graph (of the nodes that it can actually reach). Write a function

```
int eccentricityOf(const Map<string, Set<string>>& graph, const string& node);
```

that returns the eccentricity of the given node in the given graph. As a hint, think about using breadth-first search.